Fuzzy Inference

- Most common fuzzy inference technique used is the Mamdani method
  - Built one of the first fuzzy systems in 1975.
- Uses four steps
  - Fuzzification of the input variables
  - Rule evaluation
  - Aggregation of the rule outputs
  - Defuzzification

Fuzzy Inference

- Example – two input, one output problem with three rules.
  - Rule 1: IF x is A3 OR y is B1 THEN z is C1
  - Rule 2: IF x is A2 AND y is B2 THEN z is C2
  - Rule 3: IF x is A1 THEN z is C3
  - Where x, y and z are linguistic variables (project funding, project staffing and risk)
  - A1, A2, A3 are linguistic values (inadequate, marginal, adequate) determined by fuzzy sets on universe of discourse X (project funding)

Fuzzy Inference

- B1, B2 (large, small) are determined by fuzzy sets on universe Y (project staffing)
- C1, C2, C3 (low, normal, high) are determined by fuzzy sets on Z (risk).

Fuzzy Inference

- Can rewrite the rules as
  - IF project funding is adequate OR project staffing is small, THEN risk is low.
  - IF project funding is marginal AND Project staffing is large, THEN risk is normal.
  - IF project funding is inadequate, THEN risk is high.

Fuzzification

- The input fuzzy sets.

Fuzzification

- The experts determine the extent to which project funding and staffing are really adequate.
  - Different systems use different crisp inputs.
  - Some can be easily measured (height, weight, distance, pressure), while others depend only on expert estimate.
Fuzzification

• Once crisp inputs are obtained, they are fuzzified against the appropriate sets.
  – Crisp input x1 (funding rated as 35% by the expert) has membership in both the inadequate and marginal sets (0.5 and 0.2).
  – Similar for y1 (rated as 60%), has degrees of fulfilment of 0.1 and 0.7 in the sets small and large.
• Each input is fuzzified over all the membership functions used by the fuzzy rules.

Rule Evaluation

• Take the fuzzified inputs and apply them to the antecedents of the fuzzy rules.
  – $\mu_{x=A1} = 0.5$, $\mu_{x=A2} = 0.2$, $\mu_{y=B1} = 0.1$, $\mu_{y=B2} = 0.7$
• If a rule has multiple antecedents, the fuzzy operator (AND, OR) is used to obtain a single number.
  – This is then applied to the consequent.

Rule Evaluation

– To evaluate disjunctions, use the OR fuzzy operation, using the classical fuzzy operation UNION
  • $\mu_{A \cup B}(x) = \max [\mu_A(x), \mu_B(x)]$
  • This operation can be customised. MATLAB offers a probabilistic OR method as well. Calculated as
  • $\mu_{A \cup B}(x) = \text{probor} [\mu_A(x), \mu_B(x)]$
    $\mu_A(X) + \mu_B(X) - \mu_A(X) \ast \mu_B(X)$

Rule Evaluation

– To evaluate conjunctions, use the AND operator or Intersection.
  • $\mu_{A \cap B}(x) = \min [\mu_A(x), \mu_B(x)]$
  • MATLAB also offers the product and
    $\mu_{A \cap B}(x) = \text{prod} [\mu_A(x), \mu_B(x)]$
    $\mu_A(X) \ast \mu_B(X)$

Rule Evaluation

• Doctors differ…
• Do different evaluation methods give different answers.
  – Yes.
• What to do?
  – You decide, depends on the application.

Rule Evaluation

• Rules again, using expert values.
• Rule 1: if x is A3 (0.0) OR y is B1 (0.1) THEN z is C1 (0.1)
  – $\mu_{z=C1}(Z) = \max [\mu_{A3}(X), \mu_{B1}(Y)] = \max [0, 0.1] = 0.1$
• Rule 2: if x is A2 (0.2) AND y is B2 (0.7) THEN Z is C2 (0.2)
  – $\mu_{z=C2}(Z) = \min [\mu_{A2}(X), \mu_{B2}(Y)] = \min [0.2, 0.7] = 0.2$
Rule Evaluation

- Now apply the result of the previous calculations to the consequent.
  - The consequent membership function is clipped or scaled to the level of the truth value of the rule antecedent.
- Clipping and Scaling describe two slightly different ways of evaluating the membership value for the consequent.

Rule Evaluation

- Clipping
  - Most common method.
  - Just cuts the consequent membership function at the level of the antecedent truth.
  - Simple method of correlation between the two parts of the rule.
  - Some information in the top of the function is lost, but the maths is quick and less complex, and the resulting aggregated surface is easier to defuzzify.

Rule Evaluation

- Scaling
  - Also called correlation product.
  - Better for preserving the shape of the fuzzy set.
  - The original membership function of the rule consequent is adjusted by multiplying all its membership degrees by the truth value of the rule antecedent.
  - Very useful and loses less information.

Aggregation of outputs

- The process of unification of the outputs of all the rules.
  - Take the membership function of the consequents (clipped or scaled) and combine them into a single fuzzy set.
  - The input is the list of clipped/scaled consequent fuzzy sets and the output is one fuzzy set for each output variable.

Defuzzification

- Final step.
- Fuzziness is useful for evaluation, but the final output still has to be a crisp number.
  - Process of arriving at a single number is defuzzification.
- Most popular method is the centroid technique.
  - Find the point where a vertical line splits the set into two equal masses.
    - Mathematically, the centre of gravity (COG)

Defuzzification

- Expressed as:
  \[ COG = \frac{\int_{a}^{b} \mu_{A}(x) \, dx}{\int_{a}^{b} \mu_{A}(x) \, dx} \]
- In theory, COG is calculated over a continuum of points.
  - In practice, a reasonable estimate is obtained by using a sample of points.
Defuzzification

- This reasonable estimate is given by
  \[
  COG = \frac{\sum_{x} \mu_A(x) x}{\sum_{x} \mu_A(x)}
  \]
- Use this to calculate the COG for the 3 rules above.

This gives a crisp output \( z_1 \) of 67.4 and can be interpreted as a 67.4% risk involved in the “fuzzy” project.